CMPT 260 Final Exam
December 19, 2005
Closed Book. 3 hours

Instructions. Please answer all questions in the exam booklet. A portion of the marks awarded will be for the style and clarity of your answer. Marks are indicated in boldface.

Marks are indicated in bold face.

Do not use a hard pencil.

1. Logic (short answer)**(15)**
   a) **(3)** What is a proof?
   b) **(4)** Give the truth table for $P \Rightarrow Q$.
   c) **(8)** State whether each of the following sentences of first order logic is a *tautology*, a *falsehood*, or *neither*.

      i. $(\forall x \forall y\ P(x,y)) \Leftrightarrow (\forall y \forall x\ P(x,y))$

      ii. $(\forall x \exists y\ P(x,y)) \Leftrightarrow (\exists y \forall x\ P(x,y))$

      iii. $(\forall x(P(x)) \vee (\forall x(Q(x)) \Rightarrow (\forall x(P(x) \vee Q(x))$

      iv. $(\forall x(P(x) \vee Q(x))) \Rightarrow (\forall x(P(x)) \vee (\forall x(Q(x)$

2. Functions (short answer – proofs not required)**(8)**
   a) **(2)** Is the function $P{:}\{1,2,3\} \rightarrow \{a,b\}$, given by $\{ (1,a), (2,b), (3,b) \}$ one-to-one, onto, both, neither, or is it not really a function?

   b) **(2)** Is the function $A{:}\{a,d\} \rightarrow \{1,2,3\}$, given by $\{ (a,1), (d,2) \}$ one-to-one, onto, both, neither, or is it not really a function?

   c) **(2)** Is the function $F{:}R \rightarrow R$ given by $F(x) = x^3 + x - 8$ one-to-one, onto, both, neither, or is it not really a function?

   d) **(2)** Is the function $F{:}R \rightarrow R$ given by $F(x) = x^2 + 3x + 2$ one-to-one, onto, both, neither, or is it not really a function?

3. Sets and Relations (short answer)**(23)**
   a) **(4)** Define *equivalence relation*.
   b) **(5)** What is the powerset of {0,1}? What is {0,1}$\times$ {0,1}?
   c) **(2)** Define equality over sets. That is, what does it mean when we say two sets are equal?
   d) **(4)** Let S = {*a,b,c,d*} and let R={(*a,b*),(*d,a*)} be a relation on *S.* Give the transitive closure of *R* and the reflexive closure of *R*.
   e) **(3)** Let R = {(1,*a*),(2,*b*),(3,*c*)} and S = {(*a*,*A*),(*a*,*B*),(*c*,*C*)}. Find $R \circ S$.
   f) **(5)** Let *P*, *Q* and *R* be sets such that $P \cup Q = P \cup R$. Is it true that $Q=R$? If your answer is yes, prove it. If your answer is no, give a counterexample.

4. **(8)** Give the truth table for
$$(S \vee \neg R) \Leftrightarrow (R \Rightarrow S).$$

5. **(10)** Prove $(D \Rightarrow E) \Rightarrow ((D \vee C) \Rightarrow (E \vee C))$.

6. **(18)** Assume that addition has been defined as it was in class, and that the basic properties of addition (associativity, commutativity, special additive properties of zero, relationship between n^+ and $n+1$) have been proved or defined.

   a) **(2)** Give the definition of multiplication.
   b) **(8)** Show that $m \cdot 1 = 1 \cdot m$. You may use *only* theorems about addition, the definition of multiplication, the theorem that $m \cdot 0 = 0 \cdot m$, and the induction hypothesis. Justify each step. Refer to theorems by name (e.g., commutativity of addition), not number.
   c) **(8)** *Using mathematical induction,* and familiar properties of arithmetic, show that if $a \neq 1$,
$$\sum_{i=0}^{n} a^i = \frac{a^{n+1}-1}{a-1}.$$
   (There is a non-inductive proof of this theorem, which is *not acceptable* as an answer here.)

7. **(12)** Do ***either*** part (a) or part (b).

   a) Write a Prolog program *thin/2*, which, given a list as its first argument, constructs a new list consisting of every second element of the list. For example:

```
?- thin( [a,b,c,d,e], F ).
F = [b,d]
?- thin( [], F).
F= []
?- thin( [1,2,3,4,5,6], A).
A = [2,4,6]
```

   b) Show that the cardinality of the even numbers E is the same as the natural numbers N by constructing a function f that is a bijection between the two sets. Prove that f is a bijection.

8. **(6)** Let A and B be the relations given by the following tables:

A	*Name*	*Number*	*Birthdate*
	Randy	123	05-04-82
	Jody	118	06-06-84
	Jamie	617	11-12-86
	Cary	622	08-09-86
	Laney	417	02-10-84

B	*Number*	*Rank*
	617	top
	622	second
	123	second

   Give

   a) The restriction of A to persons born *strictly after* 1984. (The last two digits of the *Birthdate* gives the year of birth.) Call this new relation C.

   b) The projection of C onto attributes *Number* and *Name*. Call the new relation *D.*

   c) The natural join of D and B.

-End of Exam-